Ex 1 $f(x) = x^3 + 3x^2 - 9x + 1$ cours : (1,5) Sujet A respect des notations (1)

a) $f'(x) = 3x^2 + 6x - 9$ (1)

b) $\Delta = b^2 - 4ac = 6^2 - 4 \times 3 \times (-9) = 36 + 108 = 144$

$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-6-\sqrt{144}}{2\times 3} = \frac{-6-12}{6} = -3$ $x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-6+\sqrt{144}}{2\times 3} = \frac{-6+12}{6} = 1$

(1)

4

x	-4		-3		1		4
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	21	$\nearrow$	28	$\searrow$	-4	$\nearrow$	77

$a = 3 > 0$

(2)

Ex 2 $f(x) = (2x-5)e^x$

a) $f = uv$ avec $u(x) = 2x - 5$ (0,5) $v(x) = e^x$ (0,5) or $(uv)' = u'v + uv'$

$u'(x) = 2$ $v'(x) = e^x$

d'où $f'(x) = 2e^x + (2x-5)e^x$ (1) $= e^x(2 + 2x - 5) = e^x \times (2x-3)$ (2)

b) $T : y = f'(0)(x-0) + f(0)$ (0,5) avec $f'(0) = e^0 \times (2\times 0 - 3) = -3$ et

$f(0) = (2\times 0 - 5)e^0 = -5$ (0,5)

3,5

d'où $T: y = -3(x-0) - 5$ soit $T: y = -3x - 5$ (1,5)

Ex 3 • $f(x) = \sqrt{3x^2+5}$

$f = \sqrt{u}$ $u(x) = 3x^2 + 5$ or $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$

$u'(x) = 6x$

d'où $f'(x) = \frac{6x}{2\sqrt{3x^2+5}} = \frac{3x}{\sqrt{3x^2+5}}$ (2)

• $g(x) = \frac{3x}{x^2+4}$

$g = \frac{u}{v}$ $u(x) = 3x$ $v(x) = x^2 + 4$ (0,5)

$u'(x) = 3$ $v'(x) = 2x$

$g'(x) = \frac{3\times(x^2+4) - 3x\times 2x}{(x^2+4)^2} = \frac{3x^2+12-6x^2}{(x^2+4)^2} = \frac{-3x^2+12}{(x^2+4)^2}$ (2)

• $p(x) = (x^2+2x)^3$

$p = u^n$ $u(x) = x^2 + 2x$ or $(u^n)' = nu'u^{n-1}$

$u'(x) = 2x + 2$

d'où $p'(x) = 3\times(2x+2)\times(x^2+2x)^2$
$= (6x+6)(x^2+2x)^2$ (2)

• $h(x) = e^{-x^2+3}$

$h = e^u$ $u(x) = -x^2 + 3$ or $(e^u)' = u'e^u$

$u'(x) = -2x$

d'où $h'(x) = -2x\,e^{-x^2+3}$ (2)

2

Ex 4 $f(x) = (v \circ u)(x) = v(u(x)) = v(e^x) = 2e^x + 3$ (1,5) définie sur $\mathbb{R}$ (0,5)

Ex1 $f(x) = x^3 - 3x^2 - 9x + 1$ cours : 1,5 **Sujet B** respect des notations 1

a) $f'(x) = 3x^2 - 6x - 9$ 1

b) $\Delta = b^2 - 4ac = (-6)^2 - 4\times 3\times(-9) = 36 + 108 = 144$

$x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} = \dfrac{-(-6)-\sqrt{144}}{2\times 3} = \dfrac{6-12}{6} = -1$ $\qquad x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-(-6)+\sqrt{144}}{2\times 3} = \dfrac{6+12}{6} = 3$ 1

/4

x	-4		-1		3		4
$f'(x)$		$+$	0	$-$	0	$+$	$a = 3 > 0$
$f(x)$	-75	$\nearrow$	6	$\searrow$	-26	$\nearrow$	-19

2

Ex2 $f(x) = (5x+2)e^x$

a) $f = uv$ avec $u(x) = 5x+2$, $u'(x) = 5$ $\quad v(x) = e^x$, $v'(x) = e^x$ $\quad$ or $(uv)' = u'v + uv'$

/3,5 d'où $f'(x) = 5e^x + (5x+2)e^x = e^x(5+5x+2) = e^x(5x+7)$ 2

b) $T : y = f'(0)(x-0) + f(0)$ avec $f'(0) = e^0\times(5\times 0+7) = 7$

$f(0) = (5\times 0+2)\times e^0 = 2$

d'où $T : y = 7(x-0) + 2$ soit $T : y = 7x + 2$ 1,5

Ex3 • $f(x) = \sqrt{5x^2+3}$

$f = \sqrt{u}$ $\quad u(x) = 5x^2+3$, $u'(x) = 10x$ $\quad$ or $(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

/8 d'où $f'(x) = \dfrac{10x}{2\sqrt{5x^2+3}} = \dfrac{5x}{\sqrt{5x^2+3}}$ 2

• $g(x) = \dfrac{5x}{x^2+1}$

$g = \dfrac{u}{v}$ $\quad u(x) = 5x$, $u'(x) = 5$ $\quad v(x) = x^2+1$, $v'(x) = 2x$

$g'(x) = \dfrac{5\times(x^2+1) - 5x\times 2x}{(x^2+1)^2} = \dfrac{5x^2+5-10x^2}{(x^2+1)^2} = \dfrac{-5x^2+5}{(x^2+1)^2}$ 2

• $p(x) = (2x+5)^4$

$p = u^n$ $\quad u(x) = 2x+5$, $u'(x) = 2$ $\quad$ or $(u^n)' = nu'u^{n-1}$

d'où $p'(x) = 4\times 2\times(2x+5)^3 = 8\times(2x+5)^3$ 2

• $h(x) = e^{-3x^2+1}$

$h = e^u$ $\quad u(x) = -3x^2+1$, $u'(x) = -6x$ $\quad (e^u)' = u'e^u$

$h'(x) = -6x\,e^{-3x^2+1}$ 2

Ex4 $f(x) = (u\circ v)(x) = u(v(x)) = u(\sqrt{x}) = 2\sqrt{x} + 3$ 1,5 définie sur $[0\,;+\infty[$ 0,5

/2